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New Class of Relativistic Solutions For Cahrged Anisotropic Fluid Sphers

By

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Abstract :

This papers presents new class of spherically symmetric solutions for charged anisotropic fluids in general relativity by using generating function. We have found expression for different physical variables.

Key Words : Anisotropic fluid, charge, sphere, generating function, energy momentum tensor.

1. INTRODUCTION

Several workers in general relativity have focused their mind in the study of anisotropic fluid spheres [2, 6, 18, 19].

In case of problems for massive objects in general relativity, the matter distribution is usually assumed to be locally isotropic. However, in the last few years theoretical studies on relativistic stellar models indicate that some massive objects may be locally anisotropic [1, 3, 8]. There are a number of interesting solutions that have provided insight into the effects of anisotropy on star parameters [4, 6, 7]. However, many of these solutions have a limited applicability to astrophysical situation since they do not satisfy certain physical restrictions usually imposed upon density and pressure viz., that the pressure should not exceed energy density (dominant energy condition) and that derivatives of pressure w.r.t. density should be less than or equal to unit. [1, 22, 23] macrocausatity conditions.

In recent years the solutions of Einstein's field equations corresponding to fluid distribution with anisotropic pressure have generated great interest among physicists (Bowers and Liang [3], Cosenza et al. [4], Herrer et al. [6], Ponce de Leon [12, 13, 14]; Bayin [1], Stewart [22], Singh and Singh [17], Maharaj and Maartens [9,10]. These solutions are relevant in the study of relativistic astrophysics as model of compact object which has anisotropic pressure (Rudeman [16]. Recently Rago [15] has presented an anisotropic solution which is a generalization of static solution of isotropic fluid sphere (Berger et al.) [2]. Singh et al. [18-20] have studied static anisotropic fluid spheres with non-uniform density and in higher dimensional space-time. The charged matter distribution problems in general relativity also have received considerable attention. Patino and Rago [11] have found some new solutions for charged fluid spheres. Singh et al. [20] have extended Gaete and Hojman's work [5] to the case of magneto fluids.

This paper provides some new class of solutions for charged anisotropic fluid spheres to general relativity by extending and using the work of Rago [15] in presence of an electromagnetic field. We have also found expression for physical variables like $p_r, p_\perp, \rho_m$ etc.

2. Field Equations and Conventions

We will consider the line element is given by

(2.1) $ds^2 = e^v dt^2 - e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$

where v and λ are functions of radial coordinate r alone. Einstein's field equation are

(2.2) $R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi(T_{ij} + E_{ij})$

Where the energy-momentum tensor T_{ij} for anisotropic fluid distribution is defined by

(2.3) $T_{ij} = (\rho_m + p_r)u_i u_j - p_\perp - p_\perp)x_i x_j$

where u^i is the fluid four – velocity vector $u^i = \delta_4^i e^{-\upsilon/2}$, x^i is unit space-like vector in the radial direction $x^i = \delta_1^i e^{-\lambda/2}$, ρ_m is the energy density of matter, p_r is the pressure in the direction of x_i and $p_\perp$ is the pressure on the two-space orthogonal to x_i.

The energy-momentum tensor of electromagnetic field is given by

(2.4) $E_{ij} = \frac{1}{4\pi}\left[g^{k\ell}F_{ik}F_{j\ell} - \frac{1}{4}g_{ij}F_{ki}F^{ki}\right]$

where F_{ij} is the electromagnetic field tensor defined in terms of the four-potential A_i as

(2.5) $E_{ij} = A_{j,i} - A_{i,j}$

The electromagnetic field equations are given by

(2.6) $F_{ijk} + F_{jk,i} + F_{ki,i} = 0$

(2.7) $F^{ij}; j = 4\pi J^i$

Here J^i is the four-current density. The combined Einstein-Maxwell equations for line element (2.1) can be expressed as

(2.8) $8\pi\rho_m + \frac{Q^2}{r^2} = e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2}$

(2.9) $8\pi\rho_t - \frac{Q^2}{r^4} = e^{-\lambda}\left(\frac{1}{r^2} + \frac{\upsilon'}{r}\right) - \frac{1}{r^2}$

(2.10) $8\pi p_\perp + \frac{Q^2}{r^4} = \frac{e^{-\lambda}}{2}\left(\upsilon'' + \frac{\upsilon'^2}{2} - \frac{\lambda'\upsilon'}{2} + \frac{\upsilon' - \lambda'}{r}\right)$

(2.11) $p_r' + (p_r + \rho_m)\frac{\upsilon'}{2} = \frac{2}{r}(p_\perp - p_r) + \frac{1}{8\pi r^4}\frac{dQ^2}{dr}$

where

(2.12) $Q(r) = 4\pi\int_0^r r^2 \rho_e dr$

is the charge within a sphere of radius r and charge density ρ_e is related to the proper charge density $\bar{\rho}_e$ by

(2.13) $\rho_e = \bar{\rho}_e e^{\lambda/2}$

Equation (2.8) can be integrated to given

$$(2.14)\quad e^{-\lambda} = 1\frac{2m(r)}{r} + \frac{Q^2}{r^2}$$

where we have introduced the mass function m(r) of the fluid distribution defined as

$$(2.15)\quad m(r) = \int_0^r (4\pi\rho_m r^2 + \frac{QQ'}{r})dr$$

By use of equation (2.11) and (2.14), from equation (2.9), we have

$$(2.16)\quad 8\pi\rho_r - \frac{Q^2}{r^4} + \frac{1}{r^2} = \left[1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right]\left[\frac{1}{r^2} - 2\{p_r' - \frac{2}{r}\right.$$

$$\left.(p_\perp - p_r) - \frac{QQ'}{4\pi r^4}\}/r(p_r + \rho_m)\right]$$

Now, we define a generating function

$$(2.17)\quad G(r) = \frac{\left[1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right]}{1 + 8\pi p_r r^2 - \frac{Q^2}{r^2}}$$

And an anisotropic function

$$(2.18)\quad W(r) = \frac{4(p_r - p_\perp)}{(\rho_m + p_r)} G(r)$$

With help of equation (2.15), (2.17) and (2.18) equation (2.16) can be written as

$$(2.19)\quad p_r' + \frac{(1 - G + w)(1 - 3g - G'r)}{Gr(1 + G - W)} p_r = \frac{QQ'}{2\pi r^4(1 + G - W)}$$

$$-\frac{(1 - G + W)}{8\pi r^3 G(1 + G - W)}\left[1 - G - G'r - (1 + G - G')\frac{Q^2}{r^2} + \frac{2QQ'}{r} G\right]$$

It is clear that for given G(r), W(r) and Q(r) as known functions of r, the linear differential equation (2.19) can be integrated to give the general solution

$$(2.20)\quad P_r = e^{-\int Bdr}[P_0 + \int Ce^{\int Bdr} dr]$$

where P_0 is an arbitrary integration constant and function B(r) and C(r) are given by

$$(2.21)\quad B(r) = \frac{(1 - 3G - G'r)(1 - G + W)}{rG(1 + G - W)}$$

$$(2.22)\quad C(r) = \frac{QQ'}{2\pi r^4(1 + G - W)} - \frac{(1 - G + W)}{8\pi r^3 g(1 + G - W)}$$

$$\left[1 - \frac{Q^2}{r^2} - \left(1 + \frac{Q^2}{r^2} - \frac{2QQ'}{r}\right)G - \left(1 - \frac{Q^2}{r^2}\right)G'r\right]$$

Once, p_r is known, the matter density ρ_m can be easily calculated from equations (2.15) and (1.17) obtaining

(2.23) $$8\pi\rho_m = \frac{1}{r^2}\left[1 - G(1 + 24\pi p_r r^2 + 8\pi p'_r r^3 - \frac{2QQ'}{r} + \frac{Q^2}{r^2}\right]$$

$$-G'\left(r + 8\pi p_r r^3 - \frac{Q^2}{r} - \frac{Q^2}{r^2}\right)$$

After obtaining p_r (r) and ρ_m (r), the tangential pressure $p_\perp$ can be found from equation (2.18)

(2.24) $$p_\perp = p_r - \frac{w(\rho_m + p_r)}{4G}$$

Finally taking into account equations (2.11) and (2.14) – (2.17) the metric coefficients can be expressed as

(2.25) $$e^{-\lambda} = G\left(1 + 8\pi p_r r^2 - \frac{Q^2}{r^2}\right)$$

(2.26) $$e^{\upsilon} = \frac{A^2}{r}\exp\left[\int \frac{dr}{rG}\right]$$

Here A^2 is also an integration constant.

3. Illustration of the Method

We should like to point out that any given function G(r) and change distribution Q(r) generate static, anisotropic spherically symmetric solutions of Einstein-Maxwell equations. For physically meaningful solution the generating function G must satisfy some general requirements. Assuming non-divergent pressure at the origin, the regularity conditions at the origin r = 0 [m(r)/r $\to 0$, $Q^2 / r^2 \to 0$, $e^{\lambda} \to 1$ as r $\to$ 0] imply that $\lim_{t\to 0} G(r) = 1$. If G = 1, W = 0 ($p_r = p_\perp$) and Q = 0 one obtains Minkowski flat space-time.

If we consider

(3.1) $$G = \frac{\left(1 - \frac{2M}{r_0} + \frac{e^2}{r^2}\right)}{\left(1 - \frac{e^2}{r^2}\right)}$$

(3.2) Q = e

(3.3) $$e^{\lambda} = \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1}$$

(3.4) $$e^{\upsilon} = \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)$$

By this way one gets, $p_0 = 0$ in equation (2.20), a corresponding vacuum Reissner – Nordstrom solution, irrespective of the choice of the anisotropic function. Any interior solution must join smoothly to Reissner – Nordstrom metric at the surface $r = r_0$ of the fluid distribution. For this requirement we must demand continuity of generating function at r = r_0.

(3.5) $$G(r_0) = G^{RN}(r_0) = \frac{\left(1 - \frac{2m}{r} + \frac{e^2}{r_0^2}\right)}{\left(1 - \frac{e^2}{r_0^2}\right)}$$

(3.6) $$Q(r_0) = e$$

Equation (3.6) indicates that the continuity of the radial electric field assuming no charge concentration at the boundary surface. One can easily see that there is no junction condition imposed on the anisotropic function. W.

If we consider that charge density is constant then equation (2.12) implies that Q(r) ~ r^2. The appropriate junction condition at r_0 yields.

(3.7) $Q(r) = e(r / r_0)^3$

Further, we assume

(3.8) $G(r) = 1 – ar^2$

and

(3.9) $W(r) = – ar^2$

Where a is a constant. This choice is also physically reasonable, because function G (r) ~ 1 as r ~ 0. The value of constant is to be calculated in order to satisfy the boundary conditions (3.5) and (3.6). Then

(3.10) $$a = \frac{2\left(\frac{M}{r_0} - \frac{e^2}{r_0^2}\right)}{(r_0 - e^2)}$$

With help of equation (3.7) – (3.9), then expression of the solutions, from equations (2.20)–(2.26), can be written as

(3.11) $$8\pi p_r = 8\pi p_0 + 6K^2 r^2$$

(3.12) $$8\pi\rho_m = 3a + 8\pi\rho_0(5ar^2 - 3) + (35ar^2 - 26)K^3 r^2$$

(3.13) $$8op_\perp = 8\pi p_0 + 6K^2 r^2 + \frac{ar^2}{4(1 - ar^2)}$$

$$[3a + 8\pi p_0(5ar^2 - 2) + (35ar^2 - 26)K^2 r^2]$$

(3.14) $$e^{-\lambda} - (1 - ar^2)(1 + 8\pi p_0 r^2 + 5K^2 r^2)$$

(3.15) $$e^{\upsilon} = \frac{A^2}{(1 - ar^2)^{1/2}}$$

where

$$K = e / r_{.0}^3$$

We will consider again the choice

(3.16) G(r) = b (constant)

(3.17) W (r) = c (constant)

(3.18) $Q = Kr^3$

Substituting these values into equations (2.20) – (2.26), we get the expressions for physical variables as

(3.19) $8\pi p_r = 8\pi p_0 r^{-D} + Vr^{-2} - NK^2r^2$

(3.20) $8\pi\rho_m = 8\pi bp_0(D-3)r^{-D} + (1-b-bv)r^{-2} - (1-b-5bN)K^2r^2$

(3.21) $8\pi p_\perp = 8\pi bp_0\left[1-\frac{C}{4b}(1+bD-3b)\right]r^{-D}$

$$+\left[V-\frac{C}{4b}(1-b)(1-v)\right]r^{-2}-\left[N-\frac{C}{4b}(1+N-b-5bN)\right]K^2r^2$$

(3.22) $e^{-\lambda} = 8\pi bp_0 r^{2-D} - (N+b)K^2r^4 + b + V$

(3.23) $e^{\upsilon} = A^2 r^{(1-b)/b}$

where

(3.24) $D = \frac{(1-3b)(1-b+c)}{b(1+b-c)}$

(3.25) $N = \frac{5b^2-18b+5bc+c+1}{5b^2-2b-5bc+c+1}$

(3.26) $V = \frac{(b-1)(1-b+c)}{b^2-6b-bc+c+1}$

4. Discussion

The above solution represents the uniformly anisotropic charged fluid distribution which is an anisotropic charged analogue of Tolman V solution with a slight change in notation (his n corresponding to $\frac{(1-b)}{2b}$. For a neutral isotropic sphere (i.e. K = 0, c = 0), Tolman's results are recovered. Out study is useful for insight into the effects of anisotropy on star parameters.

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